

What is Subject Content Knowledge in Mathematics? On Student Teachers' Competence, Confidence, Attitudes and Beliefs in Relation to Teaching Mathematics

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ABSTRACT

This paper builds on the findings of recent studies into the levels of mathematical competence and confidence of primary student teachers. The data from these studies were based on the use of an online assessment tool and surveys of the students who participated in its use. The findings highlighted that students' subject knowledge was often lacking when assessed using the online assessment. It was also found that those students possessing more advanced mathematics qualifications were less likely to display competence in primary mathematics and that their confidence levels in the subject were lower than predicted. The reasons for such findings are discussed in relation to beliefs about the nature of mathematics and associated attitudes towards it as a subject. This analysis is based on the consideration of opposing views of the nature of the subject which we identify as "mathematical fallibilism" on the one hand and "mathematical fundamentalism" on the other. Based on this analysis, a survey tool was designed and administered to 148 student teachers in the Autumn of 2010 in order to explore their attitudes and beliefs in relation to

mathematics. The results of this follow-up study of attitudes and beliefs are discussed in relation to the earlier findings on confidence and competence.

Key words: mathematics content knowledge, mathematics attitudes and beliefs, online assessment, teacher education, teaching mathematics

Introduction

The importance of the teachers' role in relation to the confidence shown by pupils was highlighted nearly thirty years ago in the Cockcroft Report which emphasised the way in which a teacher in every lesson conveys, even unconsciously, a message about mathematics which will influence the pupil's attitude. The studies carried out by Henderson and Rodrigues (2008) and Henderson (2010) highlighted issues related to the confidence and competence of a significant proportion of students teachers of mathematics and the findings from these studies are discussed in the first part of this paper. These findings led us to consider the importance of teachers' beliefs about the nature of mathematics and also drew attention to the contested nature of mathematics itself as a discipline e.g. a strict mathematical formalism, as discussed by Lakatos (1976) in contrast to a more informal and fallibilistic view. This discussion on the nature of mathematics forms the next section of the paper. In particular we saw the former as being distorted into a form of "mathematical fundamentalism" that can be characterised by absolutism, dogma, strict procedures, rule following and right and wrong answers. As a result of this analysis we became interested in exploring the extent to which student teachers bring with them attitudes based on such views and this forms the focus of the next stage of the paper. Finally we discuss the findings from these two phases of study and consider some of the implications for the teaching and learning of mathematics.

Student teachers' competence and confidence in teaching mathematics

Initial teacher education seeks to produce teachers whose pedagogical skills are such that they can create effective learning situations that will improve understanding in their pupils. In relation to mathematics, Ma (1999) argues that the type of pedagogical content knowledge (PCK) (Shulman 1986) necessary to achieve this is only possible if deep, broad and thorough subject matter knowledge (SMK) exists (Shulman 1986). It would seem obvious to state that in order to be able to teach a subject effectively knowledge of that subject is a pre-requisite. Anecdotally we may know teachers who are excellent mathematicians but whose teaching skills leave a lot to be desired. The converse of this is that some teachers whose mathematical skills are not well developed can still teach the subject effectively. There is evidence to refute this; teachers' knowledge of a topic influences the questions they ask their pupils (Bennett, Carré and Dunne 1993); Simon and Brown (1996) found that "gaps in subject knowledge undermine the common rationalisation of teachers' authority in the classroom" (cited in Brown, Askew, Baker, Denvir and Millett 2001); feelings of inadequacy over SMK have been shown to lead to an over-reliance on commercial schemes (Millett and Johnson 1996); Rowland, Martyn, Barber and Heal (2000) reported that student primary teachers without strong subject knowledge are likely to perform poorly in mathematics teaching when assessed, even towards the end of their training. As Shulman (1986) asked,

"What prices are paid when the teacher's subject matter competence is itself compromised by deficiencies of prior education or ability?" (p.8).

Against this background an initiative was taken to address the limitations of SMK on the part of student primary teachers at a UK University through the creation of an instrument known as the Online Maths Assessment (OMA) which aimed to identify and address deficiencies in mathematics content knowledge and skills. It also aimed to raise students' awareness of the mathematics topics in which proficiency was sought, as well as the levels of competence required for the primary school. The first study to come from this (Henderson & Rodrigues 2008) reported that first year student

primary teachers' levels of competence and confidence could not be guaranteed by the mathematics entrance level qualifications for primary teaching that currently exist. What is more it was found that those students' competence levels in primary mathematics fell as they proceeded along the mathematics qualifications hierarchy, with 73% who possessed the Scottish Higher scoring below 80% in the assessment compared to 61% of those with the lower Standard Grade/Intermediate 2 qualification. This resonates with Bennett and Carré's (1993, p.61) work which reported that "there is virtually nothing to distinguish [the teaching performance of] mathematicians and other in teaching mathematics" and Askew, Brown, Rhodes, Johnson and Wiliam (1997) who found that teachers did not have to hold advanced mathematics qualifications to be effective. When confidence was considered in the Dundee study (Henderson and Rodrigues 2008) it was found that three quarters of the Standard Grade/Intermediate 2 group and half of the Higher group reported having little or no confidence in their skills.

When the OMA was rolled out across all four years of the undergraduate Bachelor of Education (BEd) programme it was apparent that there was a pattern to the levels of engagement displayed by the students which placed them in one of the following four groups¹:

- A. Those who had 1-2 attempts and stopped at or just over the threshold (80% or 83%);
- B. Those who had multiple attempts (3 or more) and stopped at or just over the threshold (80% or 83%);
- C. Those who had multiple attempts and scored more than 5% over the threshold (87% or over);
- D. Those who had 1-2 attempts and scored more than 5% over the threshold (87% or over).

In order to investigate this further a second study was conducted with a sample of student primary teachers ($n = 80$) (Henderson 2010) in an attempt to answer the following research questions:

¹ It should be noted that the number of questions in the OMA is the reason that the percentages jump from 80% - 83% - 87%, rather than in single percentiles.

1. Why do some students stop at a tutor-determined, pre-set threshold and others improve on it?
2. What factors are associated with differences in how students engage with the OMA?

The findings from this study revealed particular group characteristics, highlighted issues related to subject knowledge and affirmed the importance of the affective dimension. Each aspect is discussed in the following sections.

Group characteristics

Ideally all mathematics teacher educators would want their student teachers to belong to Group C. The students in this group, despite sometimes having mixed experiences of mathematics when they attended school, were motivated enough to continue attempting the OMA until they were satisfied with their above-threshold scores. In addition, they reported good or growing levels of confidence in their mathematics ability.

The Group A students did not appreciate that achieving 80% in the OMA highlighted the fact that there was 20% of required knowledge and skills they did not possess. They made excuses about why they stopped when they did, often citing that as they had reached the threshold, they had done what they were asked to do. Complacent is perhaps a better description of this group. In her study of 432 post-graduate student primary teachers, Goulding (2003) confirmed the existence of such a group following an audit of their mathematics knowledge and skills.

The Group B students needed several attempts to get to the threshold but then stopped. The fact that they needed multiple attempts should have suggested to them that their knowledge and skills required further revision. However, they, like the students in Group A, made excuses for not continuing beyond the threshold. The Group C students, on the other hand, were committed to achieving the best results they could and invested time and effort into so doing. Unlike students in the other groups, those in Group

C did not use the excuses of shortage of time or stress for not engaging with the OMA. Instead they identified with the importance of improving their knowledge and skills.

Group D students had achieved high scores in a minimum number of attempts so it was expected that students in this group would have high levels of confidence given that their competence was not in doubt. It was therefore interesting to discover that these students were quite mixed, confirming a previous study, in which high levels of competence were also not always accompanied by confident attitudes (Henderson and Rodrigues 2008). Assuming that students who achieve a high score on a mathematics competence test are secure or confident in their knowledge or skills cannot be taken for granted. It does appear to be the case, however, that a positive attitude and engagement in improving OMA scores can lead to increased levels of confidence and motivation by students to improve further.

Subject knowledge

This study was undertaken after it had become apparent that there were concerns about students' mathematics knowledge and skills. In their study Hill, Rowan and Ball (2005) reported that knowledgeable teachers may provide better mathematical explanations, produce better representations of concepts, hear and understand their pupils' responses the better to direct them in their learning, and have more understanding of how the different branches of mathematics connect. In other words, these knowledgeable teachers have made the transformation from SMK to successful PCK.

Instead of being able to rely upon a sound knowledge base, it has become increasingly necessary in teacher education to address the content as well as the teaching of the content, yet there is little time in such courses to make up for any deficit in content knowledge. The onus is then placed on the student to address any deficiencies in SMK and, as this study has shown, the motivation for students to do so can be mixed. It appears to be the student's own realisation that spurs them from the conscious incompetence stage to address inadequacies in their knowledge so that they can reach the conscious competence stage (Luft and Ingham 1955, cited in Perkin

1999). This appears to be the effect that engagement with the OMA had on the Group C students. This study has shown that the use of a tool such the OMA can go some way towards improving SMK, with the added advantage that this improvement may lead to increased levels of confidence, resulting in increased motivation to engage further with it.

Affect

In addressing the knowledge, beliefs and attitudes of mathematics teachers, Ernest (1989) distinguished between the cognitive outcome of knowledge and the affective outcomes of attitudes and beliefs in teacher education, going on to say that teacher educators must differentiate between the two. As posited by Szydlik, Szydlik and Benson (2003) the prevailing beliefs of student teachers about mathematics are hindering positive and productive mathematics learning. This study drew attention to link between the acquisition and improvement of mathematics knowledge and the attitudes and beliefs that student teachers hold. It can be posited that the Group C students' continued engagement with the OMA led to success which in turn led to further attempts. So while this study set out to improve the SMK of student primary teachers, it became apparent that the reasons for the different patterns of engagement with the OMA were influenced by affective factors as much as cognitive ones, although improvement in the cognitive, that is, increases in OMA scores, often led to improved confidence and attitudes.

Accordingly it was decided to focus attention in the second phase of this study on student teachers' beliefs and attitudes towards the subject and on the nature of mathematics itself. In our view such affective aspects have been neglected in the process of reform over recent decades. Furthermore we argue that these findings point towards the importance of teachers' beliefs about the nature of mathematics in the formation of such attitudes. In particular we are interested in previous research which shows that differing conceptions on the nature of mathematics have an influence on the ways in which teachers and mathematicians approach the teaching and development of mathematics (Thompson 1984; Cooney 1985). This brings us to questions related to the nature of mathematics itself.

On the nature of mathematics

If we look to the history of the development of the discipline of mathematics, it can be seen that the nature of the subject itself has long been contested. This has profound implications for school mathematics - for example is it an abstract subject for an elite or should mathematics be for all? In the analysis contained in his seminal text Lakatos (1976, p 5) distinguishes between the deductivist approach and the heuristic approach which he describes as "the logic of proofs and refutations". With regard to the former it is argued that Euclidean methodology developed a certain obligatory style of presentation which is described as deductivist style:

This style starts with a painstakingly stated list of axioms, lemmas and/or definitions. The axioms and definitions often look artificial and mystifyingly complicated. One is never told how these complications arose. The list of axioms and definitions is followed by carefully worded theorems. These are loaded with heavy-going conditions; it seems impossible that anyone should ever have guessed them. The theorem is followed by proof.

Lakatos (1976, p 142)

Mathematics is compared with a conjuring act according to this "Euclidian ritual" and the student is obliged to accept this without asking questions about the underlying assumptions. In this deductivist style, under which all propositions are true and all inferences valid, mathematics is presented as an ever-increasing set of eternal, immutable truths. We argue that it is not simply the dominating influence of this deductivist approach which is a main problem for the teaching and learning of mathematics in schools today but rather the way in which this has become distorted into a form of fundamentalism that is akin to religious fundamentalism.

We argue that such a fundamentalism promotes an authoritarian view of mathematics, which hides the struggle and adventure involved. In turn such authoritarianism is the very antithesis of the conditions needed to foster independent and critical thinking. An alternative perspective, which we describe as mathematical fallibilism (Lakatos 1976), argues for a view of mathematics as

human activity and that it is this human mathematical activity that produces mathematics. However when it is presented in textbooks this product of human activity "alienates itself" (ibid, p146) from the very human activity, which produced it. The mathematics educator Geoff Giles (Giles, 1982, p37) used the expression which captures the essence in very vivid for us of dead geometry "entombed in text books". This stands in stark contrast, for example, with the present day opportunities afforded by the use of dynamic geometry software to enable students to independently study the invariant (unchanging) relationships between points, lines and circles, forming their own conjectures and testing them out visually, which is the very essence of geometry. Such diametrically opposing viewpoints about the nature of mathematics are captured in Table 1 below:

Table 1. The contested nature of mathematics

Mathematical fundamentalism	Mathematical fallibilism
<ul style="list-style-type: none"> ▪ Infallible and authoritarian ▪ Dogmatic and absolutist ▪ Irrefutable and certain ▪ Strict procedures ▪ Rule following ▪ Right and wrong answers ▪ High stakes testing ▪ Boring ▪ De-motivating ▪ Fear and anxiety ▪ Alienation from the subject itself 	<ul style="list-style-type: none"> ▪ Fallible and liberating ▪ Growth and change ▪ Refutable and uncertain ▪ Multiple solutions ▪ Creative reasoning ▪ Errors and mistakes ▪ Evaluation & self assessment ▪ Engaging ▪ Motivating ▪ Enjoyment and fulfilment ▪ A creative human activity

Accordingly this table was used as the basis of the questions in a survey of student primary teachers in relation to their attitudes and beliefs towards and about mathematics in the Autumn Term of 2010.

On student teachers' beliefs about and attitudes towards mathematics

Methodology

Primary education programmes at the University of Dundee are highly over subscribed, with students drawn from across Scotland and the rest of the UK, so it can safely be claimed students attracted to the teacher education courses are of the highest calibre. The purpose of the study was to investigate student primary teachers' attitudes to and beliefs about mathematics with a view to adding to the body of knowledge already growing from previous studies in the Dundee project about competence and confidence in mathematics for teaching. The online survey was conducted with students enrolled on Years 1 to 4 of a BEd Honours degree programme and a Post-Graduate Diploma in Education (Primary) (PGDEP) course during the week beginning 12th September 2010. It was completed by 148 students from a population of 388, representing a response rate of 38%.

Instrument

The questionnaire was adapted from Aiken's Revised Math Attitude Scale (1974) by using the terminology from Table 1. A series of statements was created and students asked to indicate their agreement or otherwise with these, on a scale of 1-5, where 1 was *strongly agree* and 5 *strongly disagree*. They were also given the opportunity to make further comment under a section entitled *Any other reflections*. The survey was created using Google's Survey Monkey tool.

Procedure

The students were sent an email which included a link to the survey and asked to complete it within a week. A second email was sent after a few days as a reminder. At the end of the week the data were downloaded and initially all data were examined together, then PGDEP separated from BEd and finally BEd 1 from BEd 4 to see if any changes had taken place over the four years of the undergraduate programme.

Findings

In order to analyse the data more closely it was decided to separate the statements into attitude and belief statements and the findings are reported under these headings below. Before carrying out a closer analysis, however, it is worth considering initial impressions from the data. What was most apparent was the core of students who held fairly fundamentalist attitudes and beliefs towards mathematics, somewhere in the region of 20–25% who see mathematics as an uninspiring, strictly rule-driven activity with little room for creativity. When the BEd programme alone is considered these figures drop to around 15–20% for students in the final year of the programme but as this represents one fifth of teachers about to enter the profession it has to remain a concern. What was also clear from the data was the number of students who would not commit to either agreeing or disagreeing with the statements and who appeared content to sit on the fence. This could have been a confidence issue and if so would only serve to increase the percentages given above, with obvious implications.

Attitudes

As the PGDEP students represent only 19% of the data collected as expected their opinions do not differ greatly in most statements from the group as a whole. The most they vary is when the statement *I am motivated to do maths when I can* is considered and only 43% of the group, compared to 55% of all respondents strongly agreed or agreed with this statement. Also 32% of this group, compared to 24% of all respondents strongly agreed or agreed with the statement *Thinking about maths makes me fearful and anxious*. Students were also asked the highest mathematics qualification they had and while 54% of PGDEP students reported having the Scottish Higher, only 10% had a higher qualification than this, indicating that the vast majority of the group had done little or no mathematics since leaving school. Their trepidation about mathematics after four years of study in other disciplines can therefore be understood.

It would be hoped that attitudes to mathematics improve as student teachers progress through an initial teacher education programme and this can be seen to be the case. Table 2 gives the comparisons

between BEd 1 and BEd 4 students' responses to the attitude statements. The data was collected using a 5 point Likert scale with 1 being strongly agree and 5 strongly disagree. For the purposes of reporting 1/2 are reported together as are 4/5.

Table 2. Attitude comparison between BEd 1 and BEd 4

Attitude statements	BEd 1			BEd 4		
	1/2	3	4/5	1/2	3	4/5
I like maths	37	34	30	61	26	13
I am good at maths	29	41	29	57	30	13
I am confident about doing maths	29	29	34	57	26	17
Maths is boring	22	27	51	9	9	83
Maths is interesting	54	27	20	83	17	0
I am not motivated to do maths unless I have to	39	22	39	17	17	65
I am motivated to do maths when I can	56	22	22	78	22	0
Thinking about maths makes me fearful and anxious	27	22	51	13	26	61
I find maths enjoyable and fulfilling	32	37	32	65	17	17
I hate maths	17	20	63	4	9	87

A great deal of emphasis is placed on the importance of positive attitudes to mathematics in the mathematics education lectures and workshops and it appears that this is effective. The different attitudes reported by BEd 4 when compared to those of BEd 1 are marked in all statements. That said there remains a small yet significant minority of around 10% who continue to have negative attitudes about the subject.

Beliefs

When the belief statements are considered, once again there is little disagreement between those reported by the group as a whole and the PGDEP students. What is most marked, and is to be expected, is that there is broad agreement between PGDEP students and BEd 1 students in the statements *Maths is a solitary activity* and *Maths is*

about right and wrong answers. These two beliefs about mathematics are possibly most indicative of the fundamentalism described previously. Interestingly 50% of PGDEP students believe that maths is a creative human activity, a figure that compares only with 48% of BEd 4 students. So while PGDEP students' beliefs tend to align with those of BEd 1, in this respect these students are more in agreement with BEd 4. This could have something to do with the fact that their own degrees included some elements of mathematics which led them to see it in a more creative light. This, of course, is speculation without further investigation.

Again it would be hoped that beliefs about mathematics are changed during teacher education courses and table 3 indicates that this is the case.

Table 3. Belief comparison between BEd 1 and BEd 4

Belief statements	BEd 1			BEd 4		
	1/2	3	4/5	1/2	3	4/5
Maths is about certain truth	37	46	17	57	39	4
Maths is about things that are uncertain	20	49	32	9	43	48
Maths is about following rules strictly	46	20	34	43	30	26
Maths is about my own creative reasoning	32	34	34	57	30	13
Maths is a solitary activity	15	27	59	0	4	96
Maths involves discussion with others	63	27	10	78	13	9
Maths is about strict procedures	29	24	46	26	39	35
Maths is about multiple solutions	41	39	20	39	39	22
Maths is about right and wrong answers	56	10	34	39	26	35
Maths involves learning from errors and mistakes	93	2	5	100	0	0
Maths is about testing	44	37	20	26	43	30
Maths involves me in evaluating my own achievements	54	29	17	52	35	13
I think that maths is a creative human activity	29	32	39	48	31	22

While there was a minority of around 10% who continued to hold negative attitudes to mathematics in the final year of a teacher education programme this figure is higher when the belief statements are analysed, at around 17%. What is most concerning for the teachers these students will go on to become is the 43% who believe *Maths is about following rules strictly*, a figure which remains roughly consistent in all four years of the programme and with PGDEP students also and the 26% who believe *Maths is about testing*.

Student voices

Students were also given the opportunity to add open comments about mathematics and many of these were about their own experiences of mathematics while at school.

... my low confidence in maths was down to teachers in my Standard Grade year who weren't helpful, encouraging or positive in their teaching.

I have always been scared of doing mental maths and did not enjoy maths at school ...

I recall the attitude of my class teacher in primary school ... if you answered a mathematical question incorrectly you were belittled in front of your classmates and made to feel very inadequate.

Fear of maths is a learned state which is the result of poor teaching.

I have always believed that the wrong mode of teaching maths can have an extremely negative impact on the perception of maths for the learner.

I have always enjoyed maths. I had a great maths teacher throughout my time in secondary and this motivated me to learn more and achieve in maths.

Discussion and implications

The results of the first phase of studies highlighted that students' subject knowledge was often lacking when assessed using the online assessment. They also highlighted the fact that those students possessing more advanced mathematics qualifications were less likely to display competence in primary mathematics and that their confidence levels in the subject were lower than predicted. The first study in the project concentrated on BEd 1 students only and found that almost 60% of students reported low levels of confidence in mathematics. The second study which included all four years of the BEd programme found that, while this figure dropped as students progressed through the programme, it was still the case that one third of students overall reported low levels of confidence. It is well documented that low levels of confidence can impact negatively on students' performance in mathematical tasks (Pajares & Miller 1994, Pajares & Miller 1995) and, perhaps more importantly, that it can play a crucial role in students' academic motivation (Pajares 1996). Hence the amount of persistence and perseverance they put into mathematics may depend on their levels of confidence and this is an area that needs to be considered in teacher education.

The findings from the second phase confirmed that there was a significant minority of students (18%) who hold negative attitudes and beliefs towards mathematics at the final stages of their studies, as defined in the contested nature of mathematics statements used in the survey. However even larger minorities can be seen to hold to quite fundamentalist views about the nature of the subject with 43% believing that mathematics is about following rules strictly and 26% who believe mathematics is about testing.

There has been a recent review of teacher education in Scotland (Donaldson 2010) which highlights that current qualifications in mathematics do not seem to provide the guaranteed levels of subject competence required for teaching. Ma (1999) stated that it was unrealistic to expect SMK to improve until mathematics education in school also improved. In reality, it would take many years for change in the school system to feed through and so the concerns about levels of mathematics competence of some student primary

teachers need to be addressed as part of teacher education. Indeed the Donaldson review (2010) recommends that prospective teachers should be able to demonstrate competence in numeracy before being accepted on teacher education programmes and that these programmes should address any weaknesses during the course. While it is disappointing that the focus in Donaldson (2010) appears to be numeracy rather than mathematics in its wider sense, it is clear that the issue of teacher competence is high on the political agenda.

What is less apparent is any move to address the affective domain of learning mathematics, despite evidence that this is as important as the cognitive domain. There appears to be attitude and belief change in the Dundee students as they progress through the four years of the BEd programme and engagement with the online assessment may be a contributing factor. It is certainly the case that this leads to increased levels of confidence. Ensuring that this change is not transitory, resulting in these students going on to teach mathematics as they were taught it (Ball 1988, Melnick & Meister 2008), often along more fundamentalist lines, may be difficult to achieve unless the nature of mathematics is thoroughly explored during teacher education. While Macnab and Payne's (2003) study found that two-thirds of Scottish BEd students' understanding of, confidence in and liking for mathematics changed for the better as a result of their course, the researchers went on to posit that these positive responses,

'may have been obtained at the expense of the development of their critical thinking about the nature of mathematical understanding and the ability to think mathematically' (p.65).

Despite this, attitude and belief change, and hence increasing confidence levels, can only be sustainable if the nature of mathematics is more thoroughly explored in teacher education, whether this be at the initial phase or as part of continuing professional development. This requires a focus on what it means to think mathematically so that a deeper understanding of the subject is achieved. Mason (2010) proposes that the three factors that influence mathematical thinking are competence in the use of mathematical enquiry

processes, confidence in handling the emotional and psychological states associated with the subject and understanding the content of mathematics. However, it is the last of these that provokes most attention when perhaps it is the first two that need more focus.

The studies described point towards the importance of sound subject knowledge and indeed without this the transformation to PCK cannot take place. They also show the impact of negative attitudes and beliefs on students' motivation to develop their mathematical knowledge. It is now timely, however, to look further at the questions raised by the studies into the importance both of teachers' beliefs and the affective dimension for student learning, as well as to explore the nature of mathematics itself. It is against this background that the authors are now involved in developing a module with the aim of supporting the continuing professional development (CPD) of teachers of mathematics at the primary level. This addresses the question of the nature of mathematics and the importance of developing mathematical thinking as opposed to the dry, procedure driven approach that is often typical of the mathematics classroom. It is intended that the module will provide the first step on a pathway to the 'reinvigoration of subject expertise', as described in the recent review of teacher education (Donaldson 2010, p.99), and give teachers access to the high quality CPD discussed therein.

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Mathematics Teachers' Self-regulated Learning Competencies

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ABSTRACT

The aim of this paper is to present a research made among 41 mathematics teachers regarding self-regulated learning of Mathematics. A questionnaire was designed and completed by these teachers, to find out their level of self-regulated learning competency and the methods they use for increasing pupils' self-regulation level. The research results show a correlation between the teachers' self-regulated competency level and their involvement in developing students' self-regulated learning skills. The results underline the necessity to develop pre-service and in-service teachers self-regulated learning, to change the Romanian mathematics curriculum in order to promote self-regulated learning, and at the same time to rethink the national Mathematics tests for including challenging problems, not only verify the use of concepts and algorithms.

Key words: self-regulated learning, mathematics education, teacher education.

Introduction

Self-regulated learning (SRL) is an academically effective form of learning, through which the learner sets goals and makes plans before starting to learn; monitors and regulates his/her cognition, motivation and behavior during the learning process; and reflects on his/her learning process (Pintrich, 1995; Pintrich, 2000; Zimmerman, 2001). Self-regulation is important in problem solving, thus developing SRL skills and teaching of mathematics are in strong relation. Many countries have changed their mathematics curriculum and adapted a problem-solving approach. This approach gives the possibility to develop SRL skills, but in the same time it necessitates to increase the learners self-regulated level (Pape & Smith, 2002).

Romanian pupils' self-regulated learning skills are around average. Marchis and Balogh (2010) made a research with secondary school pupils' (10-15 years old, 5th-8th grades) studying their interest for mathematics and their self-regulated learning skills as self-efficacy, self-judgement, and self-reaction. Their findings are the following: only one third of the respondents like mathematics, because the others don't see the links between mathematics and their everyday life; almost half of the pupils have low self-efficacy and more than half of the respondents feel a high level of anxiety. In the case of high-school pupils (14-19 years old, 9th-12th grades) Marchis (2010) has concluded that only one third of the respondents think that mathematics is useful in everyday life and their future career; a very low percentage of the pupils' set goals for learning mathematics; about one fourth of the respondents analyze correctly the task and have a correct task difficulty perception; pupils' self-efficacy and self-control is low, but they have a high self-judgement level. Pupils' mathematical results are in strong correlation with their interest to study mathematics, their task analysis and self-control skills, and their task difficulty perception. Thus there is a need to motivate pupils for learning mathematics and to develop their self-regulated learning skills. The mathematics teacher has an important role in this process.

The aim of this research is to study mathematics teachers' self-regulation during problem solving by evaluating competencies as self-efficacy and self-control; and investigate if teachers' SRL skills are in correlation with their expectations on pupils' problem-solving behavior. Recommendations for increasing teachers' and pupils' self-regulated level while learning mathematics is formulated in the conclusions.

1. Self-regulation in mathematics learning

Self-regulated learners are metacognitively, motivationally and behaviorally active participants in their own learning process (Zimmerman, 1986).

SRL has three phases: forethought, performance control, and self-reflection phase.

From cognitive point of view during the **forethought phase** the learner analyses the task: activates his/her prior content and metacognitive knowledge, sets the goals of the learning process, and plans the strategy to be used. When solving a mathematical problem, the analysis of the task includes the understanding of the problem; identifying the given data, the relations between these data, and the requirements of the problem; recalling prior knowledge related with the problem (definitions; theorems, algorithms, strategies which could be used, similar worked examples). Motivationally goal-orientation, self-efficacy, perception of task difficulty, and activation of the interest to perform the task are important in this phase. A study in the field of goal-orientation reported important relation between different goals and self-regulation (Tanner & Jones, 2003). Mastery goal-orientation is also positively related with the use of cognitive and self-regulated strategies (Pintrich, 1999). Self-efficacy has a key role in SRL. It refers to perceptions about one's capabilities to organize and implement actions in order to reach the desired performance level (Bandura, 1997). Self-efficacy is student's judgments about their ability to successfully complete a task, as well as students' confidence in his/her skills to perform the task (Pintrich et al., 1993). "People's beliefs in their efficacy influence the choices they make, their